MOMENTS OF THE COLLISION INTEGRAL FOR MAXWELLIAN MOLECULES

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The Maxwellian model for interaction between molecules is often used in solving various problems in the kinetic theory of gases (the repulsive force g is inversely proportional to the fifth power of the distance between molecules).

This interaction law simplifies the structure of the collision integral considerably, and, in particular, enables one to calculate the moments of this integral in a finite form, which must be known when solving problems of the kinetic theory of gases by the methods of moments. Below, we give moments of the collision integral of the fourth and fifth order for Maxwellian molecules.

The methods of calculating the moments and formulas for the lower moments may be in papers [1–3].

We introduce the following symbols for the density, the nth moment of the distribution function and the pressure, respectively:

$$\rho = mn = m \int F(\mathbf{x}, \xi, t)\, d\xi,$$

$$M_{i_1 \ldots i_n} = m \int c_{i_1} \ldots c_{i_n} \; F(\mathbf{x}, \xi, t)\, d\xi, \quad M_{nn} = 3p .$$

Here F(x, ξ, t) is the distribution function, m is the mass of the molecules,

$$c_i = \xi_i - u_i, \qquad u_i = \frac{1}{n} \int \xi_i F(\mathbf{x}, \xi, t)\, d\xi .$$

Let

$$C(Q) = \int_{\infty} d\xi \int_{\infty} d\xi_1 \int_0^{\infty} b\, db \int_0^{2\pi} d\varepsilon\, \delta Q F F_1 V$$

$$(\delta Q = Q_1' + Q' - Q_1 - Q, \quad Q_1' = Q(\xi_1')).$$

In this notation we have

$$m^2 C(c_i c_j c_k c_l) = -\sqrt{G/2m}\, A_2 [7\rho M_{ijkl} + 3M_{(ij}M_{kl)} -$$
$$- 3\rho M_{mm(ij}\delta_{kl)} - 9pM_{(ij}\delta_{kl)}] + \sqrt{G/2m}\; {}^1/_{16} A_4 [35\rho M_{ijkl} +$$
$$+ 105 M_{(ij}M_{kl)} - 30\rho M_{mm(ij}\delta_{kl)} - 90pM_{(ij}\delta_{kl)} -$$
$$- 60 M_{m(i}\delta_{jk}M_{l)m} + 3\rho M_{mmnn}\delta_{(ij}\delta_{kl)} + 27p^2\delta_{(ij}\delta_{kl)} +$$
$$+ 6M_{mn}M_{mn}\delta_{(ij}\delta_{kl)}], \qquad (1)$$

$$m^2 C(c_i c_j c_k c_l c_m) = -\sqrt{G/2m}\, A_2 [10\rho M_{ijklm} + 5M_{(ij}M_{klm)} -$$
$$- 5\rho M_{nn(ijk}\delta_{lm)} - 15pM_{(ijk}\delta_{lm)}] + \sqrt{G/2m}\; {}^5/_{32} A_4 [35\rho M_{ijklm} +$$
$$+ 70 M_{(ij}M_{klm)} - 30\rho M_{nn(ijk}\delta_{lm)} -$$
$$- 90pM_{(ijk}\delta_{lm)} - 60M_{n(i}\delta_{jk}M_{lm)n} + 30M_{nn(i}M_{jk}\delta_{lm)} +$$
$$+ 3M_{ppnn(i}\delta_{jk}\delta_{lm)} + 18pM_{nn(i}\delta_{jk}\delta_{lm)} +$$
$$+ 12M_{pn}M_{pn(i}\delta_{jk}\delta_{lm)} - 12M_{ppn}M_{n(i}\delta_{jk}\delta_{lm)}]. \qquad (2)$$

Here the brackets surrounding certain groups of s indices indicate a sum over s! permutations of these indices, divided by s!, and recurring indices indicate summation over these indices. The constants A_2 and A_4 are determined by the expressions

$$A_2 = \sqrt{m/2G}\, V\pi \int_0^{\infty} \sin^2 2\theta b\, db = 1.3694,$$

$$A_4 = \sqrt{m/2G}\, V\pi \int_0^{\infty} \sin^4 2\theta b\, db = 0.8649 .$$

The quantity G is the coefficient in the interaction law $g = Gr^{-5}$, and the collision parameters b and $2\theta - \pi$ are the impact parameter and angle of inclination of the relative velocity V, respectively. For convenience we shall introduce similar formulas, expressed in terms of spherical moments and coefficients of the Hermite distribution functions.

Using the symbols of paper [2], formulas (1) and (2) may be written in the form

$$mC(c^4) = -\frac{2}{3}\frac{n}{\rho} B_2 \left[\rho P_{4|} - 15p^2 + P_{ij}P_{ij}\right], \qquad (3)$$

$$mC(c^2 Y_{ij}) =$$
$$= -\frac{7}{6}\frac{n}{\rho} B_2 \left\{\rho P_{2|ij} - pP_{ij} + \frac{4}{7}\left[P_{ik}P_{kj} - \frac{1}{3} P_{kl}P_{kl}\delta_{ij}\right]\right\}, \qquad (4)$$

$$mC(Y_{ijkl}) = -\frac{1}{4} n (6B_2 + B_4) P_{ijkl} + \frac{3n}{4\rho}(2B_2 - B_4)\left[P_{(ij}P_{kl)} -\right.$$
$$\left.- \frac{4}{7} P_{m(i}\delta_{jk}P_{l)m} + \frac{2}{35} P_{mn}P_{mn}\delta_{(ij}\delta_{kl)}\right]. \qquad (5)$$

$$mC(c^4 Y_i) =$$
$$= -\frac{n}{\rho} B_2 \left[\rho P_{4|i} - \frac{28}{3} ph_i + \frac{2}{3} P_{ijk}P_{jk} + \frac{28}{15} P_{ij}h_j\right] \qquad (6)$$

$$mC(c^2 Y_{ijk}) = -\frac{n}{14}(19B_2 + 2B_4) P_{2|ijk} -$$
$$- \frac{9n}{7\rho}(B_2 - B_4) pP_{ijk} - \frac{3n}{14\rho}(8B_2 - B_4) \times$$
$$\times \left[P_{l(i}P_{jk)l} - \frac{2}{5} P_{lm}P_{lm(i}\delta_{jk)}\right] +$$
$$+ \frac{9n}{35\rho}(13B_2 - 6B_4)\left[h_{(i}P_{jk)} - \frac{2}{5} h_l P_{l(i}\delta_{jk)}\right], \qquad (7)$$

$$mC(Y_{ijklm}) = -\frac{5n}{8}(2B_2 + B_4) P_{ijklm} +$$
$$+ \frac{5n}{84\rho}(2B_2 - B_4)\left[21 P_{(ij}P_{klm)} +\right.$$
$$\left.+ 2P_{pn}P_{pn(i}\delta_{jk}\delta_{lm)} - 14 P_{n(i}\delta_{jk}P_{lm)n}\right]. \qquad (8)$$

Here the parameters B_2 and B_4 are associated with A_2, A_4 in the following manner:

$$B_2 = 3A_2 \sqrt{G/2m} = 3\sqrt{G/2m}\, 1.369,$$

$$B_4 = \sqrt{G/2m}\,(10A_2 - {}^{35}/_4 A_4) = 2\sqrt{G/2m}\, 3.06^3.$$

Using the symbols of paper [3] the same formulas take the form

$$mJ_{ijkl}^{(4)} = -B_1\rho\,[14a_{ijkl}^{(4)} - 6a_{mm(ij}^{(4)}\delta_{kl)} + 6a_{(ij}^{(2)}a_{kl)}^{(2)}] +$$
$$+ B_3\rho\,[35a_{ijkl}^{(4)} - 30a_{mm(ij}^{(4)}\delta_{kl)} + 3a_{nnmm}^{(4)}\delta_{(ij}\delta_{kl)} +$$
$$+ 105a_{(ij}^{(2)}a_{kl)}^{(2)} - 60a_{m(i}^{(2)}\delta_{jk}a_{l)m}^{(2)} + 6a_{nm}^{(2)}a_{nm}^{(2)}\delta_{(ij}\delta_{kl)}]. \qquad (9)$$

$$mJ_{ijklm}^{(5)} = -10B_1\rho\,[2a_{ijklm}^{(5)} + a_{(ij}^{(2)}a_{klm)}^{(3)} - a_{nn(ijk}^{(5)}\delta_{lm)}] +$$
$$+ {}^1/_2 B_3\rho\,[175a_{ijklm}^{(5)} + 350a_{(ij}^{(2)}a_{klm)}^{(3)} - 150a_{nn(ijk}^{(5)}\delta_{lm)} -$$
$$- 300a_{n(i}^{(2)}\delta_{jk}a_{lm)n}^{(3)} + 150a_{nn(i}^{(3)}a_{jk}^{(2)}\delta_{lm)} + 15a_{ppnn(i}^{(5)}\delta_{jk}\delta_{lm)} +$$
$$+ 60a_{pn}^{(2)}a_{pn(i}^{(3)}\delta_{jk}\delta_{lm)} - 60a_{ppn}^{(3)}a_{n(i}^{(2)}\delta_{jk}\delta_{lm)}]. \qquad (10)$$

Here

$$B_1 = {}^1/_2 m A_2 \sqrt{G/2m}, \qquad B_3 = {}^1/_{16} m A_4 \sqrt{G/2m} .$$

Formulas (3)–(7) can be found in [2] (a mistake was made in formula (6) of that paper, and the coefficient B_4 was incorrectly calculated).

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